Gough - Stuart platform leg length, valid volume calculation

[Ref. 'The Mathematics of the Stewart Platform' unknown author, &
http://memememememememe.me/post/stewart-platform-math/]

Mid z position of platform $z_{mid} := 22.19mm$

Platform displacement

Roll (rx) $\varphi := 0deg$

Pitch (ry) $\theta := 0deg$

Yaw (rz) $\psi := 0deg$

roll rotation matrix $R_x := \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & -\sin(\varphi) \\ 0 & \sin(\varphi) & \cos(\varphi) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Pitch rotation matrix $R_y := \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Yaw rotation matrix $R_z := \begin{pmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Combined rotation matrix $R_B := R_x \cdot R_y \cdot R_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Base anchor point co-ordinates (local)

leg:	0	1	2	3	4	5	axis
	4	26	22	−22	−26	−4	x
	27.7	−10.39	−17.32	−17.32	−10.39	27.7	y
	0	0	0	0	0	0	z

$b := \begin{pmatrix} 4 & 26 & 22 & -22 & -26 & -4 \\ 27.7 & -10.39 & -17.32 & -17.32 & -10.39 & 27.7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot mm$

Platform anchor point co-ordinates (local)

$p := \begin{pmatrix} 5.5 & 7.5 & 2 & -2 & -7.5 & -5.5 \\ 5.48 & 2.02 & -7.5 & -7.5 & 2.02 & 5.48 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot mm$

point of interest offset from platform coordinate

$offset := \begin{pmatrix} 8 \\ 0 \\ 10.3 \end{pmatrix} \cdot mm$

< relative to the actual position of interest on the platform or the end of the item being actuated. This then compensates for the translation effect of rotation.

leg mid length $leg_{mid} := 31.44mm$

leg length limits $leg_{max} := 1.24mm$

$leg_{min} := -1.24mm$

Derived function for populating contour chart; positive displacement

$$
\begin{aligned}
&\text{Check}(x, y) := \\
&\quad \text{temp\_k} \leftarrow 0 \\
&\quad \text{for } k \in 0..30 \\
&\quad\quad \text{vec}_{\text{leg}} \leftarrow \left| \begin{array}{l} \text{for } n \in 0..5 \\ \quad l^{\langle n \rangle} \leftarrow \begin{pmatrix} x \cdot mm + \text{offset}_0 \\ y \cdot mm + \text{offset}_1 \\ \frac{k}{10} \cdot mm + z_{\text{mid}} + \text{offset}_2 \end{pmatrix} + R_B \cdot \left(p^{\langle n \rangle} - \text{offset} \right) - b^{\langle n \rangle} \\ \text{vec}_{\text{leg}} \leftarrow l \end{array} \right. \\
&\quad\quad \text{leg}_{\text{len}} \leftarrow \left| \begin{array}{l} \text{for } n \in 0..5 \\ \quad l_{\text{len}}{}^{\langle n \rangle} \leftarrow \sqrt{\left(\text{vec}_{\text{leg}_{0,n}}\right)^2 + \left(\text{vec}_{\text{leg}_{1,n}}\right)^2 + \left(\text{vec}_{\text{leg}_{2,n}}\right)^2} \\ \text{leg}_{\text{len}} \leftarrow l_{\text{len}} - \text{leg}_{\text{mid}} \end{array} \right. \\
&\quad\quad \text{temp\_k} \leftarrow \frac{k}{10} \text{ if } \max\left(\text{leg}_{\text{len}}\right) < \text{leg}_{\max} \wedge \min\left(\text{leg}_{\text{len}}\right) > \text{leg}_{\min} \\
&\quad \text{return temp\_k}
\end{aligned}
$$

Derived function for populating contour chart; negative displacement

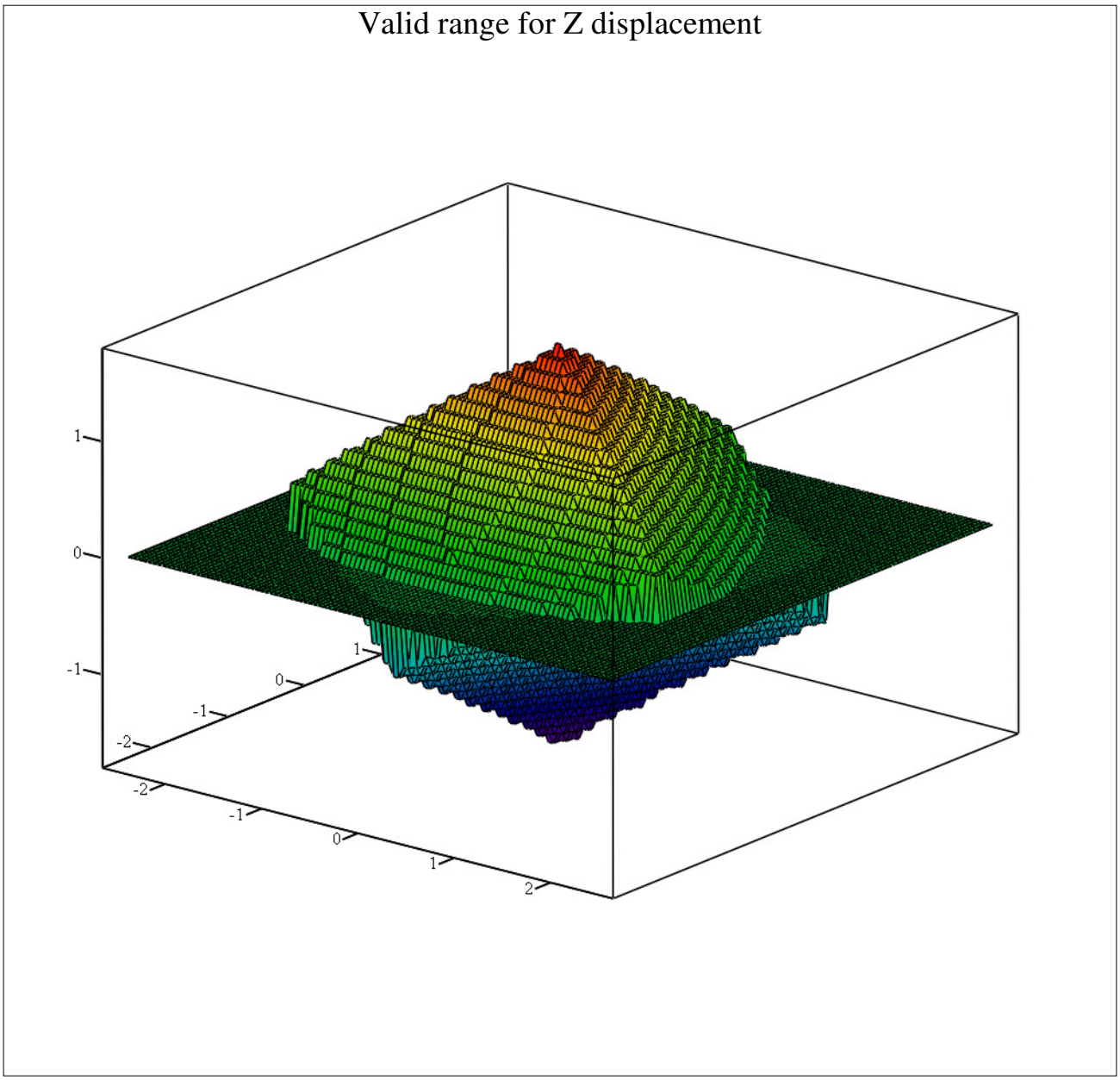
$$
\begin{aligned}
&\text{Check\_neg}(x, y) := \\
&\quad \text{temp\_k} \leftarrow 0 \\
&\quad \text{for } k \in 0..30 \\
&\quad\quad \text{vec}_{\text{leg}} \leftarrow \left| \begin{array}{l} \text{for } n \in 0..5 \\ \quad l^{\langle n \rangle} \leftarrow \begin{pmatrix} x \cdot mm + \text{offset}_0 \\ y \cdot mm + \text{offset}_1 \\ \frac{-k}{10} \cdot mm + z_{\text{mid}} + \text{offset}_2 \end{pmatrix} + R_B \cdot \left(p^{\langle n \rangle} - \text{offset} \right) - b^{\langle n \rangle} \\ \text{vec}_{\text{leg}} \leftarrow l \end{array} \right. \\
&\quad\quad \text{leg}_{\text{len}} \leftarrow \left| \begin{array}{l} \text{for } n \in 0..5 \\ \quad l_{\text{len}}{}^{\langle n \rangle} \leftarrow \sqrt{\left(\text{vec}_{\text{leg}_{0,n}}\right)^2 + \left(\text{vec}_{\text{leg}_{1,n}}\right)^2 + \left(\text{vec}_{\text{leg}_{2,n}}\right)^2} \\ \text{leg}_{\text{len}} \leftarrow l_{\text{len}} - \text{leg}_{\text{mid}} \end{array} \right. \\
&\quad\quad \text{temp\_k} \leftarrow \frac{-k}{10} \text{ if } \max\left(\text{leg}_{\text{len}}\right) < \text{leg}_{\max} \wedge \min\left(\text{leg}_{\text{len}}\right) > \text{leg}_{\min} \\
&\quad \text{return temp\_k}
\end{aligned}
$$

$\text{range} := 2.5 \text{ mm}$ $\text{num\_points} := 100$

build a grid of values:

$F := \text{CreateMesh}(\text{Check}, -\text{range}, \text{range}, -\text{range}, \text{range}, \text{num\_points}, \text{num\_points})$

$F\_neg := \text{CreateMesh}(\text{Check\_neg}, -\text{range}, \text{range}, -\text{range}, \text{range}, \text{num\_points}, \text{num\_points})$



Notes: The valid scope for the point of interest (for this arrangmenet), traces out a mildly inflated cube (6 sides). becuse of the limitations in graphing the corners near the zero plane look odd, but can be infered from the surfaces above and below the zero plane..

This plot is purely displacement, with no rotation, rotation cases are presented below.

Platform displacement Roll (rx) $\varphi := 5\,deg$ Pitch (ry) $\theta := 0\,deg$ Yaw (rz) $\psi := 0\,deg$

$$R_x := \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & -\sin(\varphi) \\ 0 & \sin(\varphi) & \cos(\varphi) \end{pmatrix} \qquad R_y := \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{pmatrix} \qquad R_z := \begin{pmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Combined rotation matrix $R_B := R_x \cdot R_y \cdot R_z$

$Check(x, y) :=$
- $temp\_k \leftarrow 0$
- for $k \in 0..30$
  - $vec_{leg} \leftarrow$ for $n \in 0..5$
    - $l^{\langle n \rangle} \leftarrow \begin{pmatrix} x \cdot mm + offset_0 \\ y \cdot mm + offset_1 \\ \frac{k}{10} \cdot mm + z_{mid} + offset_2 \end{pmatrix} + R_B \cdot \left(p^{\langle n \rangle} - offset\right) - b^{\langle n \rangle}$
    - $vec_{leg} \leftarrow l$
  - $leg_{len} \leftarrow$ for $n \in 0..5$
    - $l_{len}^{\langle n \rangle} \leftarrow \sqrt{\left(vec_{leg_{0,n}}\right)^2 + \left(vec_{leg_{1,n}}\right)^2 + \left(vec_{leg_{2,n}}\right)^2}$
    - $leg_{len} \leftarrow l_{len} - leg_{mid}$
  - $temp\_k \leftarrow \frac{k}{10}$ if $\max(leg_{len}) < leg_{max} \wedge \min(leg_{len}) > leg_{min}$
- return $temp\_k$

$Check\_neg(x, y) :=$
- $temp\_k \leftarrow 0$
- for $k \in 0..30$
  - $vec_{leg} \leftarrow$ for $n \in 0..5$
    - $l^{\langle n \rangle} \leftarrow \begin{pmatrix} x \cdot mm + offset_0 \\ y \cdot mm + offset_1 \\ \frac{-k}{10} \cdot mm + z_{mid} + offset_2 \end{pmatrix} + R_B \cdot \left(p^{\langle n \rangle} - offset\right) - b^{\langle n \rangle}$
    - $vec_{leg} \leftarrow l$
  - $leg_{len} \leftarrow$ for $n \in 0..5$
    - $l_{len}^{\langle n \rangle} \leftarrow \sqrt{\left(vec_{leg_{0,n}}\right)^2 + \left(vec_{leg_{1,n}}\right)^2 + \left(vec_{leg_{2,n}}\right)^2}$
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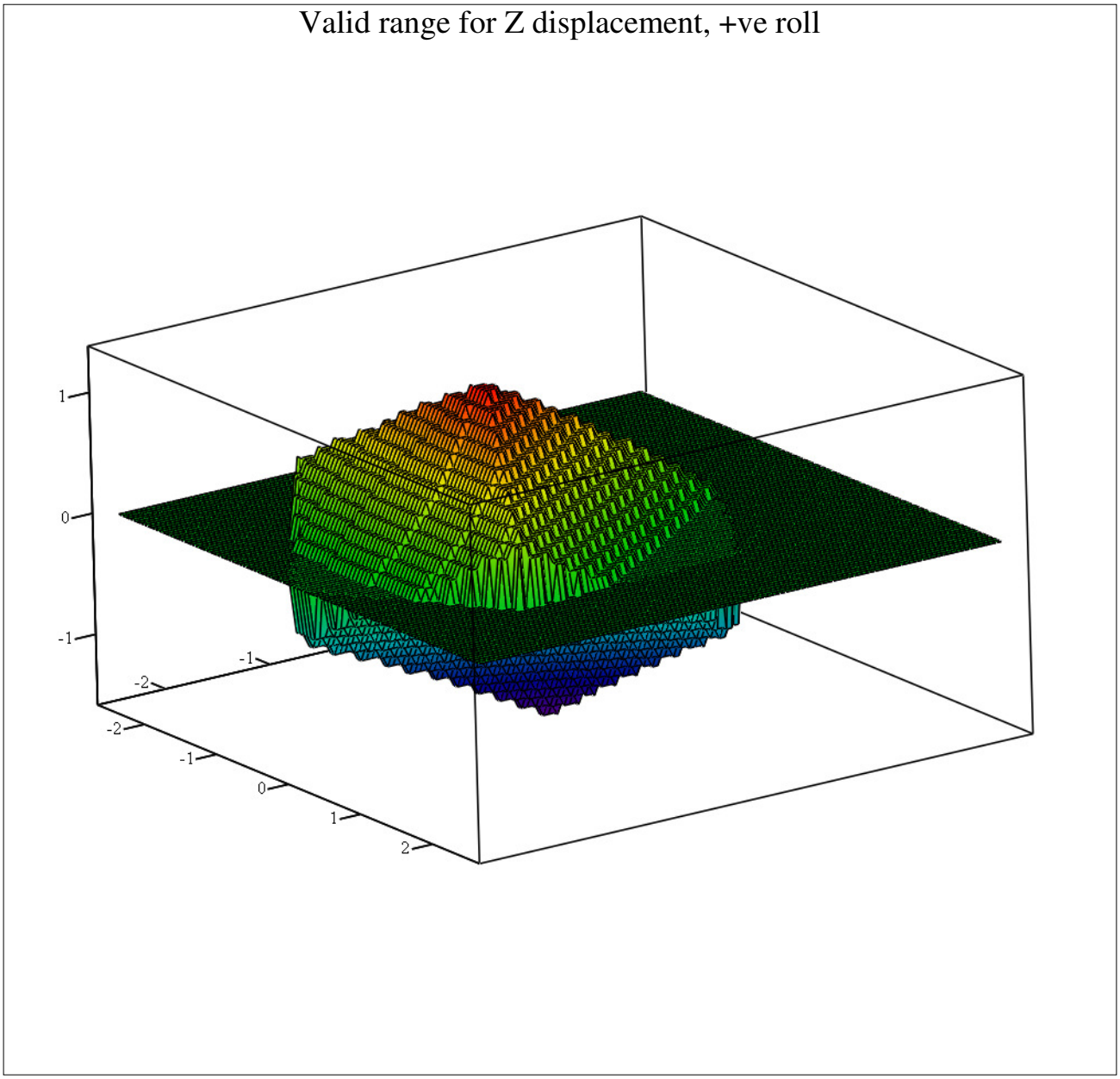
  - $temp\_k \leftarrow \frac{-k}{10}$ if $\max(leg_{len}) < leg_{max} \wedge \min(leg_{len}) > leg_{min}$
- return $temp\_k$

$range := 2.5\ mm$ $num\_points := 100$

build a grid of values:

$F := CreateMesh(Check, -range, range, -range, range, num\_points, num\_points)$

$F\_neg := CreateMesh(Check\_neg, -range, range, -range, range, num\_points, num\_points)$



With the application of angular displacement the resulting valid translational volume is thus restricted. Depending on the particular rotation the translation is restricted in different ways. Different axis rotations are plotted below.

Platform displacement Roll (rx) $\varphi := -5\,deg$ Pitch (ry) $\theta := 0\,deg$ Yaw (rz) $\psi := 0\,deg$

$$R_x := \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & -\sin(\varphi) \\ 0 & \sin(\varphi) & \cos(\varphi) \end{pmatrix} \qquad R_y := \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{pmatrix} \qquad R_z := \begin{pmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Combined rotation matrix $R_B := R_x \cdot R_y \cdot R_z$

$$\begin{aligned}
&Check(x, y) := \\
&\quad temp_k \leftarrow 0 \\
&\quad \text{for } k \in 0..30 \\
&\quad\quad vec_{leg} \leftarrow \text{for } n \in 0..5 \\
&\quad\quad\quad l^{\langle n \rangle} \leftarrow \begin{pmatrix} x \cdot mm + offset_0 \\ y \cdot mm + offset_1 \\ \frac{k}{10} \cdot mm + z_{mid} + offset_2 \end{pmatrix} + R_B \cdot \left(p^{\langle n \rangle} - offset\right) - b^{\langle n \rangle} \\
&\quad\quad\quad vec_{leg} \leftarrow l \\
&\quad\quad leg_{len} \leftarrow \text{for } n \in 0..5 \\
&\quad\quad\quad l_{len}{}^{\langle n \rangle} \leftarrow \sqrt{\left(vec_{leg_{0,n}}\right)^2 + \left(vec_{leg_{1,n}}\right)^2 + \left(vec_{leg_{2,n}}\right)^2} \\
&\quad\quad\quad leg_{len} \leftarrow l_{len} - leg_{mid} \\
&\quad\quad temp_k \leftarrow \frac{k}{10} \text{ if } \max\left(leg_{len}\right) < leg_{max} \wedge \min\left(leg_{len}\right) > leg_{min} \\
&\quad \text{return } temp_k
\end{aligned}$$

$$\begin{aligned}
&Check_neg(x, y) := \\
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&\quad \text{return } temp_k
\end{aligned}$$

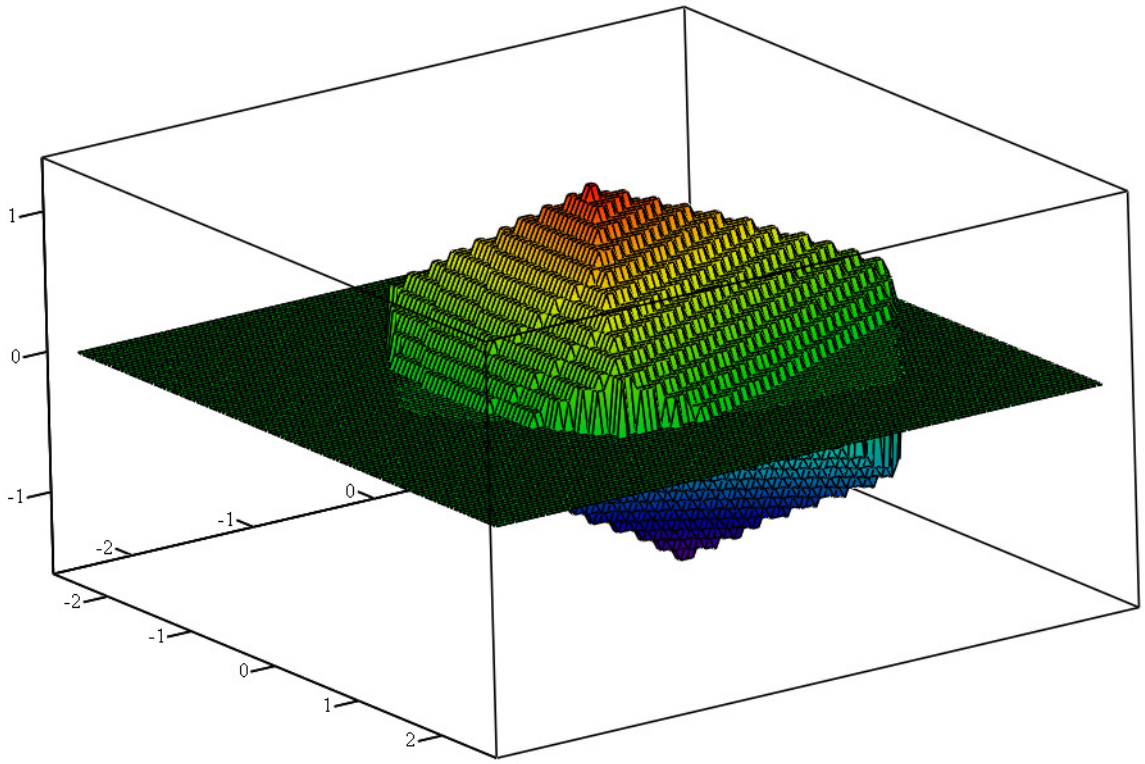
$range := 2.5\ mm$ $num\_points := 100$

build a grid of values:

$F := CreateMesh(Check, -range, range, -range, range, num\_points, num\_points)$

$F\_neg := CreateMesh(Check\_neg, -range, range, -range, range, num\_points, num\_points)$

Valid range for Z displacement, -ve roll



Platform displacement Roll (rx) $\varphi := 0deg$ Pitch (ry) $\theta := 5deg$ Yaw (rz) $\psi := 0deg$

$$R_x := \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & -\sin(\varphi) \\ 0 & \sin(\varphi) & \cos(\varphi) \end{pmatrix} \qquad R_y := \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{pmatrix} \qquad R_z := \begin{pmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Combined rotation matrix $R_B := R_x \cdot R_y \cdot R_z$

$$\begin{aligned}
&Check(x, y) := \\
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&\quad\quad\quad leg_{len} \leftarrow l_{len} - leg_{mid} \\
&\quad\quad temp_k \leftarrow \frac{k}{10} \text{ if } \max\left(leg_{len}\right) < leg_{max} \wedge \min\left(leg_{len}\right) > leg_{min} \\
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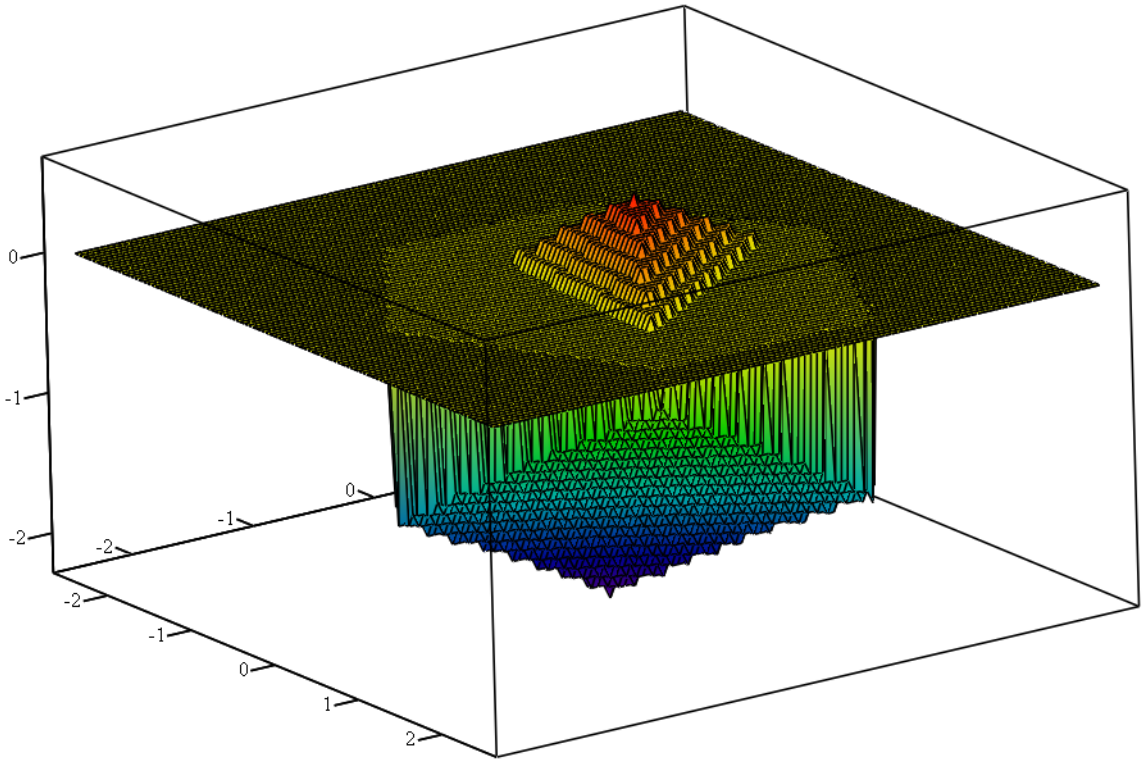
&\quad\quad\quad vec_{leg} \leftarrow l \\
&\quad\quad leg_{len} \leftarrow \text{for } n \in 0..5 \\
&\quad\quad\quad l_{len}{}^{\langle n \rangle} \leftarrow \sqrt{\left(vec_{leg_{0,n}}\right)^2 + \left(vec_{leg_{1,n}}\right)^2 + \left(vec_{leg_{2,n}}\right)^2} \\
&\quad\quad\quad leg_{len} \leftarrow l_{len} - leg_{mid} \\
&\quad\quad temp_k \leftarrow \frac{-k}{10} \text{ if } \max\left(leg_{len}\right) < leg_{max} \wedge \min\left(leg_{len}\right) > leg_{min} \\
&\quad return\ temp_k
\end{aligned}$$

$range := 2.5\ mm$ $num\_points := 100$

build a grid of values: $F := CreateMesh(Check, -range, range, -range, range, num\_points, num\_points)$

$F\_neg := CreateMesh(Check\_neg, -range, range, -range, range, num\_points, num\_points)$

Valid range for Z displacement, +ve pitch



Platform displacement Roll (rx) $\varphi := 0\,deg$ Pitch (ry) $\theta := -5\,deg$ Yaw (rz) $\psi := 0\,deg$

$$R_x := \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & -\sin(\varphi) \\ 0 & \sin(\varphi) & \cos(\varphi) \end{pmatrix} \qquad R_y := \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{pmatrix} \qquad R_z := \begin{pmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Combined rotation matrix $R_B := R_x \cdot R_y \cdot R_z$

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&\quad \text{for } k \in 0..30 \\
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&\quad \text{return } temp_k
\end{aligned}$$

$range := 2.5\ mm$ $num\_points := 100$

build a grid of values:

$F := CreateMesh(Check, -range, range, -range, range, num\_points, num\_points)$

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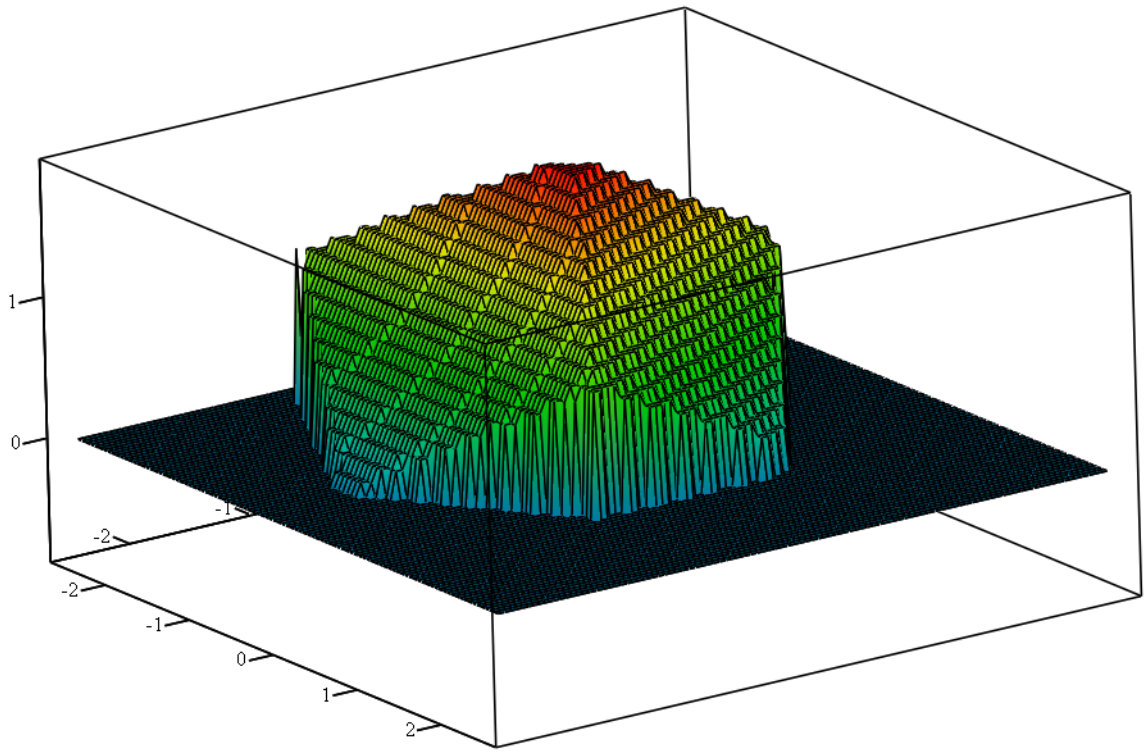
Valid range for Z displacement, -pitch



Platform displacement Roll (rx) $\varphi := 0\,deg$ Pitch (ry) $\theta := 0\,deg$ Yaw (rz) $\psi := 5\,deg$

$$R_x := \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & -\sin(\varphi) \\ 0 & \sin(\varphi) & \cos(\varphi) \end{pmatrix} \qquad R_y := \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{pmatrix} \qquad R_z := \begin{pmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Combined rotation matrix $R_B := R_x \cdot R_y \cdot R_z$

```
Check(x, y) :=
    temp_k ← 0
    for k ∈ 0..30
        vec_leg ← | for n ∈ 0..5
                  |     l^⟨n⟩ ← [ x·mm + offset_0 ; y·mm + offset_1 ; (k/10)·mm + z_mid + offset_2 ] + R_B·(p^⟨n⟩ − offset) − b^⟨n⟩
                  |     vec_leg ← l
        leg_len ← | for n ∈ 0..5
                  |     l_len^⟨n⟩ ← sqrt( (vec_leg_0,n)^2 + (vec_leg_1,n)^2 + (vec_leg_2,n)^2 )
                  |     leg_len ← l_len − leg_mid
        temp_k ← k/10  if max(leg_len) < leg_max ∧ min(leg_len) > leg_min
    return temp_k
```

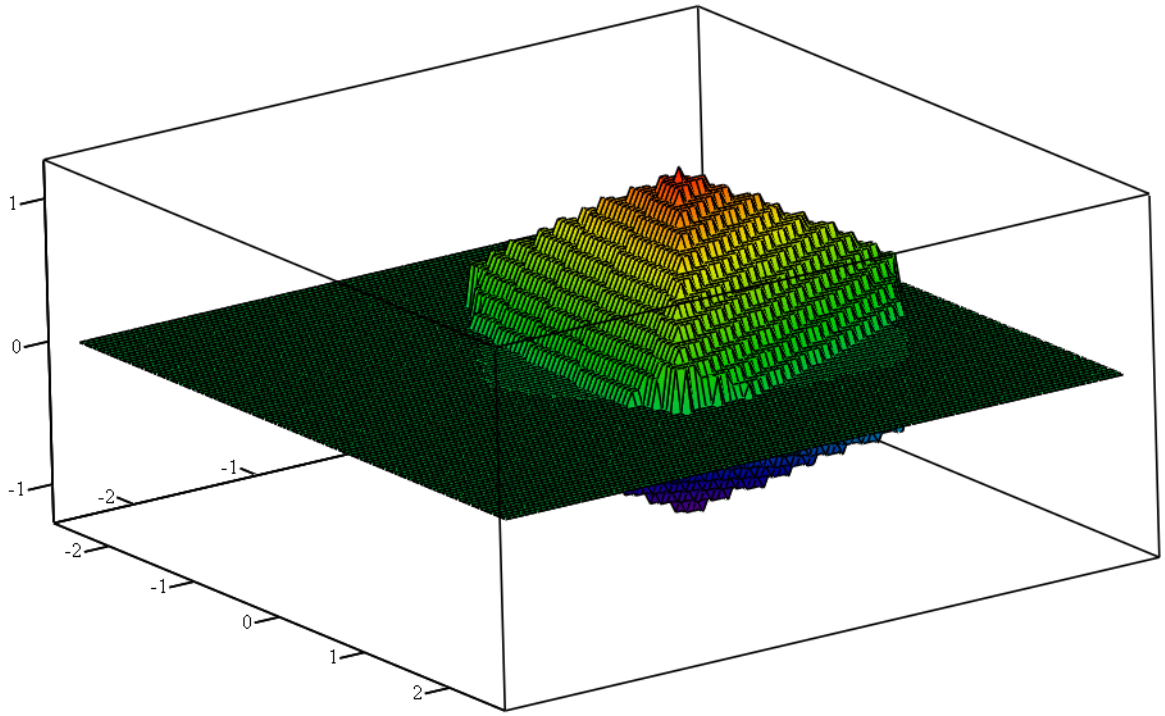
```
Check_neg(x, y) :=
    temp_k ← 0
    for k ∈ 0..30
        vec_leg ← | for n ∈ 0..5
                  |     l^⟨n⟩ ← [ x·mm + offset_0 ; y·mm + offset_1 ; (−k/10)·mm + z_mid + offset_2 ] + R_B·(p^⟨n⟩ − offset) − b^⟨n⟩
                  |     vec_leg ← l
        leg_len ← | for n ∈ 0..5
                  |     l_len^⟨n⟩ ← sqrt( (vec_leg_0,n)^2 + (vec_leg_1,n)^2 + (vec_leg_2,n)^2 )
                  |     leg_len ← l_len − leg_mid
        temp_k ← −k/10  if max(leg_len) < leg_max ∧ min(leg_len) > leg_min
    return temp_k
```

$range := 2.5\ mm$ $num\_points := 100$

build a grid of values: $F := CreateMesh(Check, -range, range, -range, range, num\_points, num\_points)$

$F\_neg := CreateMesh(Check\_neg, -range, range, -range, range, num\_points, num\_points)$

Valid range for Z displacement, +ve yaw



Platform displacement Roll (rx) $\varphi := 0\,deg$ Pitch (ry) $\theta := 0\,deg$ Yaw (rz) $\psi := -5\,deg$

$$R_x := \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & -\sin(\varphi) \\ 0 & \sin(\varphi) & \cos(\varphi) \end{pmatrix} \qquad R_y := \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{pmatrix} \qquad R_z := \begin{pmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Combined rotation matrix $R_B := R_x \cdot R_y \cdot R_z$

$$\begin{aligned}
&Check(x, y) := \\
&\quad temp_k \leftarrow 0 \\
&\quad \text{for } k \in 0..30 \\
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&\quad\quad\quad vec_{leg} \leftarrow l \\
&\quad\quad leg_{len} \leftarrow \text{for } n \in 0..5 \\
&\quad\quad\quad l_{len}{}^{\langle n \rangle} \leftarrow \sqrt{\left(vec_{leg_{0,n}}\right)^2 + \left(vec_{leg_{1,n}}\right)^2 + \left(vec_{leg_{2,n}}\right)^2} \\
&\quad\quad\quad leg_{len} \leftarrow l_{len} - leg_{mid} \\
&\quad\quad temp_k \leftarrow \frac{k}{10} \text{ if } \max\left(leg_{len}\right) < leg_{max} \wedge \min\left(leg_{len}\right) > leg_{min} \\
&\quad \text{return } temp_k
\end{aligned}$$

$$\begin{aligned}
&Check_neg(x, y) := \\
&\quad temp_k \leftarrow 0 \\
&\quad \text{for } k \in 0..30 \\
&\quad\quad vec_{leg} \leftarrow \text{for } n \in 0..5 \\
&\quad\quad\quad l^{\langle n \rangle} \leftarrow \begin{pmatrix} x \cdot mm + offset_0 \\ y \cdot mm + offset_1 \\ \frac{-k}{10} \cdot mm + z_{mid} + offset_2 \end{pmatrix} + R_B \cdot \left(p^{\langle n \rangle} - offset\right) - b^{\langle n \rangle} \\
&\quad\quad\quad vec_{leg} \leftarrow l \\
&\quad\quad leg_{len} \leftarrow \text{for } n \in 0..5 \\
&\quad\quad\quad l_{len}{}^{\langle n \rangle} \leftarrow \sqrt{\left(vec_{leg_{0,n}}\right)^2 + \left(vec_{leg_{1,n}}\right)^2 + \left(vec_{leg_{2,n}}\right)^2} \\
&\quad\quad\quad leg_{len} \leftarrow l_{len} - leg_{mid} \\
&\quad\quad temp_k \leftarrow \frac{-k}{10} \text{ if } \max\left(leg_{len}\right) < leg_{max} \wedge \min\left(leg_{len}\right) > leg_{min} \\
&\quad \text{return } temp_k
\end{aligned}$$

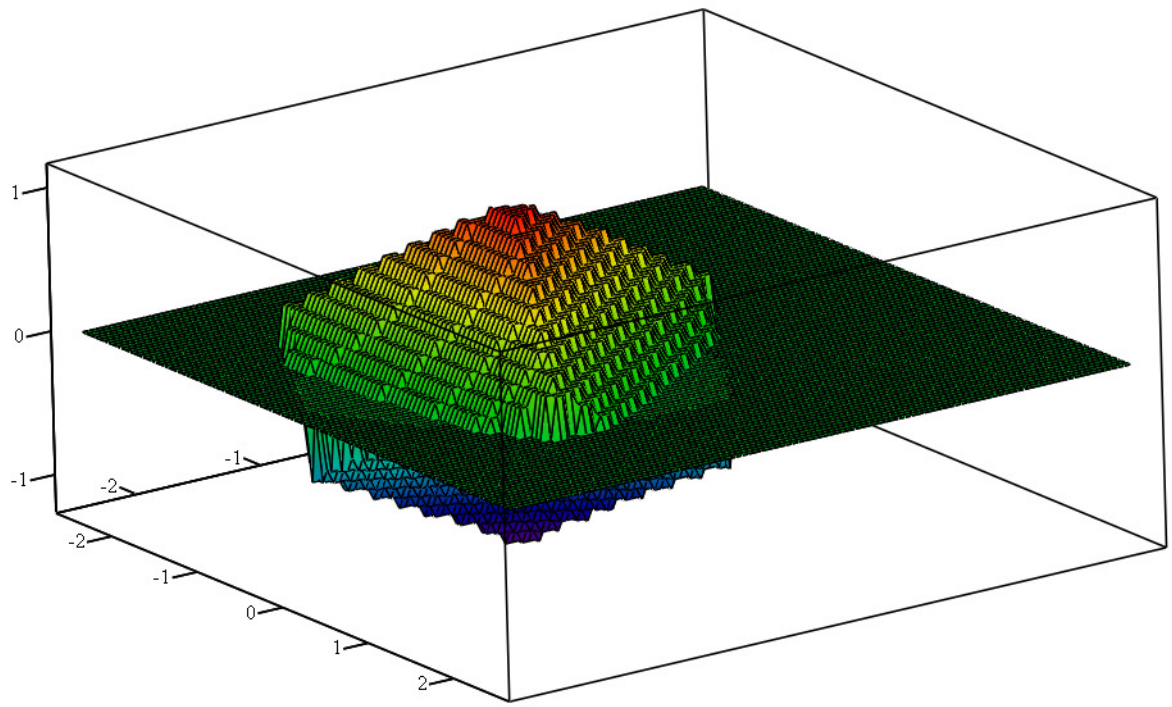
$range := 2.5\ mm$ $num\_points := 100$

build a grid of values:

$F := CreateMesh(Check, -range, range, -range, range, num\_points, num\_points)$

$F\_neg := CreateMesh(Check\_neg, -range, range, -range, range, num\_points, num\_points)$

Valid range for Z displacement, -ve yaw



Platform displacement Roll (rx) $\varphi := -5\,\text{deg}$ Pitch (ry) $\theta := -5\,\text{deg}$ Yaw (rz) $\psi := -5\,\text{deg}$

$$R_x := \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & -\sin(\varphi) \\ 0 & \sin(\varphi) & \cos(\varphi) \end{pmatrix} \qquad R_y := \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{pmatrix} \qquad R_z := \begin{pmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Combined rotation matrix $R_B := R_x \cdot R_y \cdot R_z$

$$\begin{aligned}
&\text{Check}(x, y) := \\
&\quad \text{temp_k} \leftarrow 0 \\
&\quad \text{for } k \in 0..30 \\
&\quad\quad \text{vec}_{leg} \leftarrow \left| \begin{array}{l} \text{for } n \in 0..5 \\ \quad l^{\langle n \rangle} \leftarrow \begin{pmatrix} x \cdot mm + \text{offset}_0 \\ y \cdot mm + \text{offset}_1 \\ \frac{k}{10} \cdot mm + z_{mid} + \text{offset}_2 \end{pmatrix} + R_B \cdot \left(p^{\langle n \rangle} - \text{offset} \right) - b^{\langle n \rangle} \\ \text{vec}_{leg} \leftarrow l \end{array} \right. \\
&\quad\quad \text{leg}_{len} \leftarrow \left| \begin{array}{l} \text{for } n \in 0..5 \\ \quad l_{len}^{\langle n \rangle} \leftarrow \sqrt{\left(\text{vec}_{leg_{0,n}}\right)^2 + \left(\text{vec}_{leg_{1,n}}\right)^2 + \left(\text{vec}_{leg_{2,n}}\right)^2} \\ \text{leg}_{len} \leftarrow l_{len} - \text{leg}_{mid} \end{array} \right. \\
&\quad\quad \text{temp_k} \leftarrow \frac{k}{10} \text{ if } \max\left(\text{leg}_{len}\right) < \text{leg}_{max} \wedge \min\left(\text{leg}_{len}\right) > \text{leg}_{min} \\
&\quad \text{return temp_k}
\end{aligned}$$

$$\begin{aligned}
&\text{Check_neg}(x, y) := \\
&\quad \text{temp_k} \leftarrow 0 \\
&\quad \text{for } k \in 0..30 \\
&\quad\quad \text{vec}_{leg} \leftarrow \left| \begin{array}{l} \text{for } n \in 0..5 \\ \quad l^{\langle n \rangle} \leftarrow \begin{pmatrix} x \cdot mm + \text{offset}_0 \\ y \cdot mm + \text{offset}_1 \\ \frac{-k}{10} \cdot mm + z_{mid} + \text{offset}_2 \end{pmatrix} + R_B \cdot \left(p^{\langle n \rangle} - \text{offset} \right) - b^{\langle n \rangle} \\ \text{vec}_{leg} \leftarrow l \end{array} \right. \\
&\quad\quad \text{leg}_{len} \leftarrow \left| \begin{array}{l} \text{for } n \in 0..5 \\ \quad l_{len}^{\langle n \rangle} \leftarrow \sqrt{\left(\text{vec}_{leg_{0,n}}\right)^2 + \left(\text{vec}_{leg_{1,n}}\right)^2 + \left(\text{vec}_{leg_{2,n}}\right)^2} \\ \text{leg}_{len} \leftarrow l_{len} - \text{leg}_{mid} \end{array} \right. \\
&\quad\quad \text{temp_k} \leftarrow \frac{-k}{10} \text{ if } \max\left(\text{leg}_{len}\right) < \text{leg}_{max} \wedge \min\left(\text{leg}_{len}\right) > \text{leg}_{min} \\
&\quad \text{return temp_k}
\end{aligned}$$

$range := 2.5\ mm$ $num\_points := 100$

build a grid of values: $F := \text{CreateMesh}(\text{Check}, -range, range, -range, range, num\_points, num\_points)$

$F\_neg := \text{CreateMesh}(\text{Check\_neg}, -range, range, -range, range, num\_points, num\_points)$

Valid range for Z displacement, -ve roll, pitch, yaw